

# Topological Relationships and their application

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## Definition

Topological relations between spatial objects have been widely recognised, implemented and used in GIS. They provide a notion of the general structure and the interactions of spatial objects. Topology avoids dealing with geometry by introducing topological primitives namely *boundary*, *interior* and *exterior*. The topological primitives allow to define approximate topological relationships between 0D (point), 1D (linestring), 2D (surface) and 3D (body) spatial objects in 0, 1, 2 and 3D space. The 9-intersection model is the well-known framework for detecting binary topological relationships [Egenhofer & Herring 1993], and is adopted by the OpenGeospatial consortium as a basic framework for implementation. Suppose two *simple* spatial objects  $A$  and  $B$  are defined in the same topological space  $X$  and their *boundary*, *interior* and *exterior* are denoted by  $\partial A, A^\circ, A^-, \partial B, B^\circ$  and  $B^-$ . The binary relationship  $R(A,B)$  between the two objects is then identified by composing all the possible set intersections of the six topological primitives, i.e.  $A^\circ \cap B^\circ$ ,  $\partial A \cap B^\circ$ ,  $A^- \cap B^\circ$ ,  $A^\circ \cap \partial B$ ,  $\partial A \cap \partial B$ ,  $A^- \cap \partial B$ ,  $A^\circ \cap B^-$ ,  $\partial A \cap B^-$  and  $A^- \cap B^-$ , and detecting empty (0) or non-empty (1) intersections. For example, if two objects have a common boundary, the intersection between the boundaries is non-empty, i.e.  $\partial A \cap \partial B = 1$ ; if they have intersecting interiors, then the intersection  $A^\circ \cap B^\circ$  is not empty, i.e.  $A^\circ \cap B^\circ = 1$ . Since each pair of intersections can have either the empty or non-empty value, different "patterns" define different relationships. The theoretical number of all the relationships that can be derived from the 9 intersections is  $2^9$ , i.e. 512 relationships, but only a small number of them (approximately 70) can be realized in reality. Following the 9-intersection model, operators to detect topological relationships are widely implemented in GIS and DBMS commercial software. The operations can be used for various purposes: checking inclusions and consistency between objects, finding neighbouring objects, establishing connectivity (path), composing constraints, etc.

## Historical Background

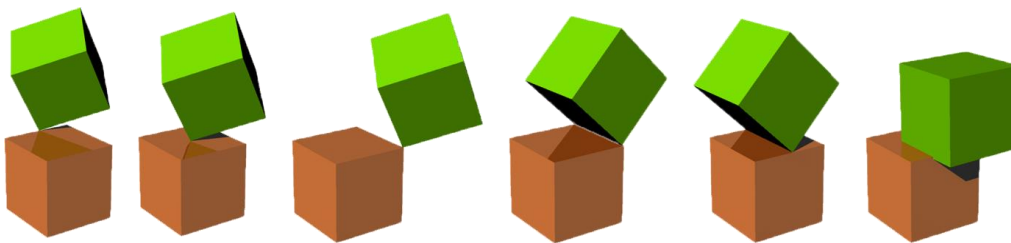
The topological relationships have gradually gained attention in the last four decades and became a major component of current spatial information applications [Breunig & Zlatanova 2011, Gröger & George, 2012]. The topic of research has shifted from issues related to the definition of a particular formalism to represent topological relationships, to implementation issues [Ellul & Haklay 2007, Borrmann & Rank 2009, Xu & Zlatanova 2013]. The research in the clarification of the relationships between spatial objects in 2D space has extended to investigation and implementation of relationships in 3D space.

The establishment of suitable framework for representation of topological relations takes relatively long time. Early work on topological relationships [e.g. Freeman 1975] does not have sufficient formalism and generality. The need for sound foundation is recognised by many researchers [e.g. Pullar & Egenhofer, 1988, Herring, 1991]. First sound models are developed on basis of point-set theory and the notion of *boundary*, *interior* and *closure* on point sets. The first, so called 4-intersection model (4IM) is presented by [Pullar & Egenhofer 1988]. It considers only point and line objects in 1D space. The topological relationships between regions are introduced by [Egenhofer & Fransoza, 1991]. The 9-intersection model (9IM) is an extension of the framework by including *exterior* of objects [Egenhofer & Herring, 1991]. A critical comparison between 4IM and 9IM have motivated the further adoption of 9IM [Egenhofer, Sharma & Mark, 1993]. The 4IM and 9IM model have triggered many studies, discussion, comparisons and criticism since then. The framework is recognised as a very powerful and sound, but also rising many questions and limitations. Topics of investigation are the number of possible relationships, the equivalent relationships, the type of objects (simple, with holes, or with broad boundaries), the naming of topological relationships.

A substantial number of studies are devoted to investigation of possible relationships. Some intersections (or a combination of intersections) between topological primitives can never occur in reality, and all the relationships that

contain these intersections (or the combination) can be securely excluded from further considerations. For example, in real world, the exterior of objects ( $A^- \cap B^-$ ) is always non-empty. This condition automatically reduces the possible relationships that can be detected with 9IM to 256. Following such approach, the possible relationships can be clarified. To eliminate all non-realizable relations, conditions, referred to as negative conditions, are composed. Several authors have presented studies on possible relationships in 2D space [Egenhofer & Herring, 1992, Kufoniya, 1995]. [De Hoop et al 1993, Egenhofer 1995] present an approach to define conditions and derive relationships in 3D space. Basically, the authors agree on the number of most of the relationships with one exception, i.e. *surface* and *surface* in 3D space. The negative conditions used by the authors vary significantly and complicates their comparison. For example, [Egenhofer & Herring, 1992] present 23 negative conditions for relationships in 2D space. To address the 3D situations, 15 more conditions are added by [van der Meij, 1992, Kufoniya, 1995], which again differ from the once presented by [Clementini & de Felice, 1997]. Furthermore most of the conditions are related to a particular configuration of objects (e.g. conditions for *linestring* and *linestring*), which leads to further duplications of the effect of some of them. [Zlatanova, 2000a] investigates all the conditions and proposes a set of 24 conditions (not considering the symmetric cases). These conditions have proven to be valid for obtaining relationship between all objects embedded in 0D, 1D, 2D and 3D space (Table 1).

While quite straight forward for simple objects, the 9IM reveals to have high complexity for objects with holes. Until recently only regions with holes in 2D have been investigated [Egenhofer, Clementini & di Felice, 1994, Schneider & Behr, 2006]. Studies in 3D space have not been presented so far. [Clementini & de Felice, 1997] address thickness of the boundary or real world objects as many objects have a boundary which cannot be approximated with a line. They present a modification of the framework for objects with broad boundaries.



**Figure 1: Equivalent topological relationships according to 9IM**

One of the most discussed topics is the fact the many relationships between real world objects appear equivalent according to 9IM (Figure 1). As the shape of the objects is not considered, it is impossible to distinguish between the type of intersection between two boundaries (0D, 1D, 2D or 3D intersection). This consequently leads to relationships which are equivalent with respect to the 9IM, but look distinctive to the human eye. Therefore several studies propose to include the dimension of the intersection in the 9IM [Clementini, di Felice P & Oosterom, 1993, Deng, Cheng Chen et al 2007, Strobl 2008, Billen & Kurata, 2008]. Dimensionally extended Nine-intersection model (DE-9IM) presented by [Strobl, 2008] has been adopted in the OGC implementations specifications in 2011.

## Scientific fundamentals

The topological relationships are defined for *topological spaces* on the basis of *set* notions [Armstrong, 1983]. A set is an aggregate of things, e.g. numbers, people, buildings, points. The things are elements of the set. A *topology* on a set  $X$  is a collection  $t$  of subsets of  $X$  called open set, satisfying:

- any union of elements of  $t$  belongs to  $t$
- any finite intersection of elements of  $t$  belongs to  $t$
- the empty set and  $X$  belong to  $t$ .

Topological space is convenient as it allows defining the continuity of space without mentioning the *distance*. Topology is derived from metric space (e.g. Euclidian), where the distance function is central. Inside a topological space, the

topological primitives, i.e. *closure*, *interior* and *boundary* of a set can be defined. If  $X$  is a topological space and  $E \subset X, K \subset X, G \subset X$ , then:

- closure is the set  $\bar{E} = \bigcap \{K \subset X \mid K \text{ is closed and } E \subset K\}$ ,
- interior is the set  $E^\circ = \bigcup \{G \subset X \mid G \text{ is open and } E \subset G\}$ ,
- boundary is the set  $\partial E = \bar{E} \cap \overline{(X - E)}$  which is a closed set.

Closure, interior and boundary of a set have an important properties [Willard 1970]:

- $E^\circ \subset E$
- $E \subset \bar{E}$
- $E$  is closed in  $X$  if and only if  $\bar{E} = E$
- $E$  is open in  $X$  if and only if  $E^\circ = E$
- $\bar{E} = E \cup \partial E$
- $E^\circ = E - \partial E$
- $X = E^\circ \cup \partial E \cup (X - E)^\circ$
- $X^\circ = X$

The topological definitions given above are not convenient for particular use (e.g. representation of geometric properties) due to the regularity of open sets in the Euclidean space, i.e. each open set of one point has the same properties as any other set. Therefore it is wiser to investigate the space around a point (or several points) and conclude that around other points it is the same. Thus the notion about a *neighbourhood* is introduced:

If  $X$  a topological space and  $x \in X$ , a *neighbourhood* of  $x$  is a set  $U$ , which contains an open set containing  $x$ , i.e.,  $x \in U^\circ$  where  $U^\circ$  is the interior of  $U$ .

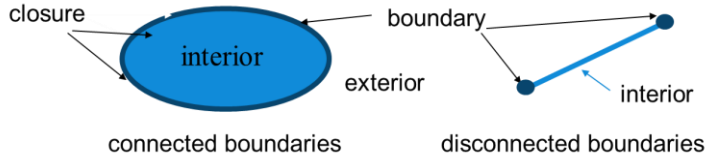
The practical importance of topology for the modelling of the spatial objects is twofold. It provides a formalism to define spatial relationships, and it allows a formal geometric description of objects. The notions of neighbourhood, interior, boundary and closure provide formalism for investigating relationships between objects without considering their shape and size. The interrelations between the topological primitives of two or more objects in their neighbourhoods define the topological relationships among objects. For example, if the neighbourhood of a point is completely inside the interior of an object, then the point is inside the object; however, if the neighbourhood is partially in the object's interior, then the point is on the boundary of the object. After applying to all of the points of two objects, this mechanism supplied information on the interrelation of two objects.

The topology can be applied to strictly define the subdivision of the space and derive the shape of objects out of it. For this purpose, an  $n$ -dimensional space (Hausdorff space) called  $n$ -manifold, in which the neighbourhood at every point is equivalent to an  $n$ -dimensional ball, is introduced. Then a subdivided manifold is defined as a pair  $(M, C)$ , where the  $M$  is manifold and  $C$  is a complex of cells. Each subdivided manifold has the following properties:

- each  $(n-1)$ -dimensional cell is shared by exactly two  $n$ -dimensional cells
- given two  $n$ -dimensional cells  $s_i$  and  $s_j$ , there exists a sequence of cells  $s_i = s_h = \dots = s_k = s_j$  such that  $s_h C_{s,k}$  is an  $(n-1)$ -dimensional face
- given any  $(n-2)$ -dimensional cell, the  $n$ - and  $(n-1)$ -cells adjacent to it can be arranged in a single connected alternating cycle.

The first property leads to a closed manifold. The closed manifolds can be orientable or non-orientable. An orientable manifold is the one, in which each two  $n$ -dimensional cells that intersect in a given  $(n-1)$ -dimensional cell induce opposite orientations on that cell. The notations on manifolds played a critical role in constructing models for computational geometry. The traditional GIS and CAD topological models (e.g. *winged-edge*, *half-wing edge*) are built on subdivisions

of 2-manifolds, i.e. vertex (0-cell), edge (1-cell) and face (2-cell). 4IM and 9IM are developed under the assumption that the definitions of spatial objects are conform to the properties of 2-manifold.



**Figure 2: 2-cell and 1-cell and their interior, boundary and exterior**

Suppose two simple spatial objects  $A$  and  $B$  defined as non-empty sets in the same topological space  $X$ , then the *boundary*, *interior*, *closure* and *exterior* will be denoted as  $\partial A, A^\circ, \bar{A}, A^-, \partial B, B^\circ, \bar{B}, B^-$ . The binary relations  $R(A, B)$  between the two objects is then defined as investigating the intersections between the boundary, interior and the exterior:

$\partial A \cap \partial B, A^\circ \cap B^\circ, \partial A \cap B^\circ, A^\circ \cap \partial B, A^- \cap B^-, A^- \cap \partial B, A^- \cap B^\circ, \partial A \cap B^-$  and  $A^\circ \cap B^-$ . The intersection (0-empty and 1-non-empty) between the six topological primitives can be represented as a matrix:

$$R(A, B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

For simplicity, instead of the matrix representation, a row representation of all the intersections is used in the following text. The intersection of the matrix above is listed in the following order:

$\partial A \cap \partial B, A^\circ \cap B^\circ, \partial A \cap B^\circ, A^\circ \cap \partial B, A^- \cap B^-, A^- \cap \partial B, A^- \cap B^\circ, \partial A \cap B^-$  and  $A^\circ \cap B^-$

For example, the relationship between objects with non-intersecting boundaries and interiors can be represented as 000011111, which can be seen as a binary number that corresponds to the decimal number 31. This number is denoted as a *decimal code* R031. This relationship corresponds to the ‘disjoint’ relationship. It is apparent that different ordering of the intersections will result in a different decimal code. This specific order is chosen, because the first four intersection represent the 4IM. The intersection  $A^- \cap B^-$  is always empty (i.e. always 0) and separates the relationships that can be detected with 4IM from 9IM.

The value of the intersections (empty, non-empty) between interior, boundary and exterior depends on following three parameters: the dimension of the objects, the dimension of the space (considered also *co-dimension* of the object) and the type of boundary (connected or disconnected). As mentioned previously, the theoretical number of relationships, i.e. 16 for 4IM and 512 for 9IM, is not possible for real world spatial objects. To eliminate the impossible relationships, negative conditions are defined. Negative conditions are composed of intersection values that are not possible (therefore the name ‘negative’) Examples of negative condition (C1, C2 and C3) are given below [Egenhofer and Herring 1992]:

C1: The exteriors of two objects always intersect.

C2: If  $A$ 's boundary intersects with  $B$ 's exterior then  $A$ 's interior intersects with  $B$ 's exterior too and vice versa.

C3:  $A$ 's boundary intersects with at least one topological primitive of  $B$  and vice versa.

The first condition restricts the intersection of the exteriors to be always non-empty, i.e.  $A^- \cap B^- = \neg \emptyset$ . This means that an empty value (or 0) can never appear in a relationship. Therefore condition C1 is represented as C1 = (-,-,-,-,0,-,-,-,-) in Table 1. After these three negative conditions, the number of possible binary relationships is reduced to 104 for spatial objects with equal dimensions (for example surface and surface) and to 160 for spatial objects with different dimensions (for example linestring and surface). The condition are often symmetric. For example, condition C3 implies that it is not

The relationships can be classified in 13 groups according to the objects they apply to:

- The set of 25 negative conditions (Table 1) is the minimal set reported currently in the literature, which identifies all possible relationships with respect to 4IM and 9IM [Zlatanova 2000b]. The negative conditions eliminate 443 and leave only 69 possible relationships. Table 1 clearly shows that the conditions for objects with equal dimension and zero co-dimension (the conditions of group 10) are very restrictive, which leaves only eight possible relationships (Figure 3, Figure 4 and Figure 5). All possible relationships are given in Table 1: Negative conditions for eliminating impossible relationships in  $\mathbb{R}$ ,  $\mathbb{R}^2$  and  $\mathbb{R}^3$

[illegible]

	C6 <sub>c</sub>	–	–	–	–	–	0	–	–	–
	C6 <sub>d</sub>	–	–	–	–	–	–	0	–	–
4	C7 <sub>a</sub>	–	0	–	0	–	–	–	–	0
	C7 <sub>b</sub>	–	0	0	–	–	–	0	–	–
5	C8 <sub>a</sub>	–	0	1	–	–	–	–	–	–
	C8 <sub>b</sub>	–	0	–	1	–	–	–	–	–
	C9 <sub>a</sub>	–	1	–	0	–	–	–	–	1
	C9 <sub>b</sub>	–	1	0	–	–	–	1	–	–
6	C10 <sub>a</sub>	1	–	–	1	–	1	–	–	–
	C10 <sub>b</sub>	1	–	1	–	–	–	–	1	–
7	C11 <sub>a</sub>	0	–	1	–	–	–	–	1	–
	C11 <sub>b</sub>	0	–	–	1	–	1	–	–	–
8	C12	0	–	–	–	–	0	–	0	–
	C13	–	0	–	–	–	0	–	0	–
	C14 <sub>a</sub>	–	–	–	–	–	–	–	0	1
	C14 <sub>b</sub>	–	–	–	–	–	0	1	–	–
9	C15 <sub>a</sub>	–	–	–	–	–	0	1	–	0
	C15 <sub>b</sub>	–	–	–	–	–	–	0	0	1
	C16 <sub>a</sub>	–	–	0	1	–	–	–	0	–
	C16 <sub>b</sub>	–	–	1	0	–	0	–	–	–
10	C17 <sub>a</sub>	–	–	0	0	–	1	–	0	–
	C17 <sub>b</sub>	–	–	0	0	–	0	–	1	–
	C18 <sub>a</sub>	–	1	1	1	–	0	–	0	–
	C18 <sub>b</sub>	1	–	1	1	–	0	–	0	–
	C19 <sub>a</sub>	1	1	1	1	–	–	–	0	–
	C19 <sub>b</sub>	1	1	1	1	–	0	–	–	–
	C20 <sub>a</sub>	–	0	–	1	–	–	0	–	–
	C20 <sub>b</sub>	–	0	1	–	–	–	–	–	0
11	C21 <sub>a</sub>	1	0	0	–	–	–	0	–	–
	C21 <sub>b</sub>	1	0	–	0	–	–	–	–	0
12	C22 <sub>a</sub>	1	–	0	–	–	1	–	0	–
	C22 <sub>b</sub>	1	–	–	0	–	0	–	1	–
13	C23 <sub>a</sub>	1	–	–	–	–	–	–	–	–
	C23 <sub>b</sub>	–	–	1	–	–	–	–	–	–
	C23 <sub>c</sub>	–	–	–	–	–	–	–	1	–
	C23 <sub>d</sub>	–	–	–	1	–	–	–	–	–
	C23 <sub>e</sub>	–	–	–	–	–	1	–	–	–
	C24 <sub>a</sub>	–	1	–	1	–	–	–	–	–
	C24 <sub>b</sub>	–	1	–	–	–	–	–	–	1
	C24 <sub>c</sub>	–	–	–	1	–	–	–	–	1
	C24 <sub>d</sub>	–	1	1	–	–	–	–	–	–
	C24 <sub>e</sub>	–	1	–	–	–	–	1	–	–
	C24 <sub>f</sub>	–	–	1	–	–	–	1	–	–
	C25 <sub>a</sub>	–	0	–	–	–	–	0	–	–
	C25 <sub>b</sub>	–	0	–	–	–	–	–	–	0

Table 2. Note that these relationships are applicable for simple spatial object, i.e. surfaces with holes or bodies with tunnels might have different relationships.

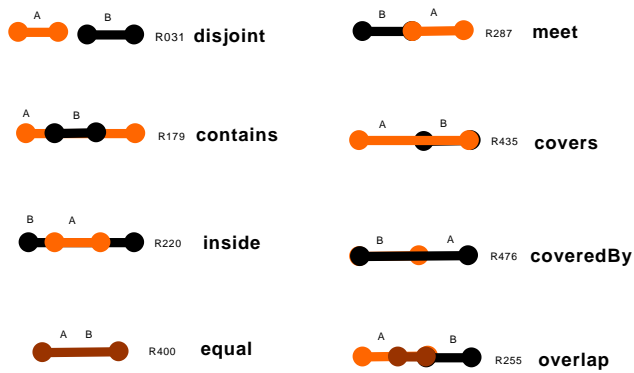


Figure 3: Relationships between lines in IR

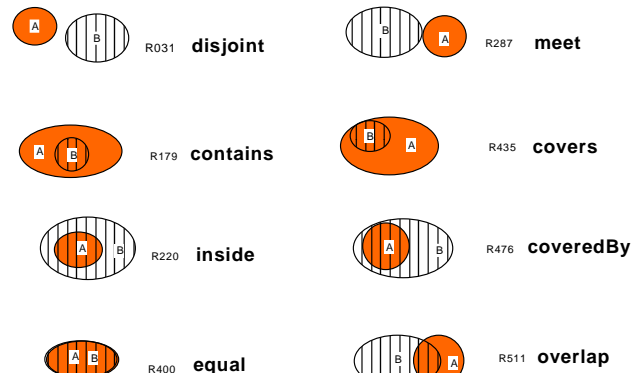


Figure 4: Relationships between surfaces IR2

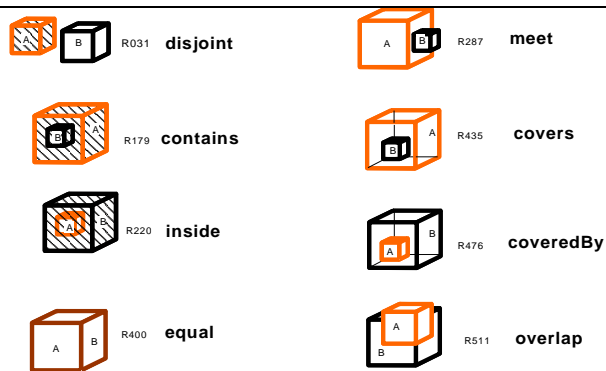


Figure 5: Relationships between bodies in IR 3 : 8 relationships

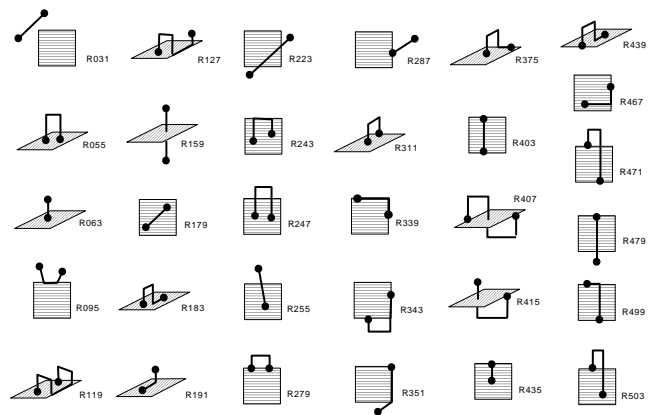


Figure 6: Surface and linestring in IR 3 : 31 relationships (19 in IR 2, face drawn as rectangle)

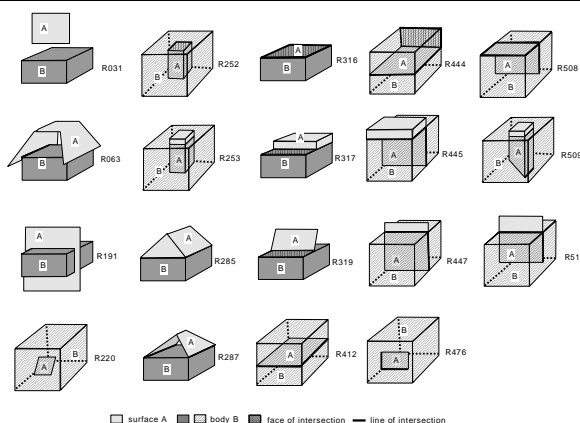


Figure 7: Surfaces and bodies in IR3. 19 relationships

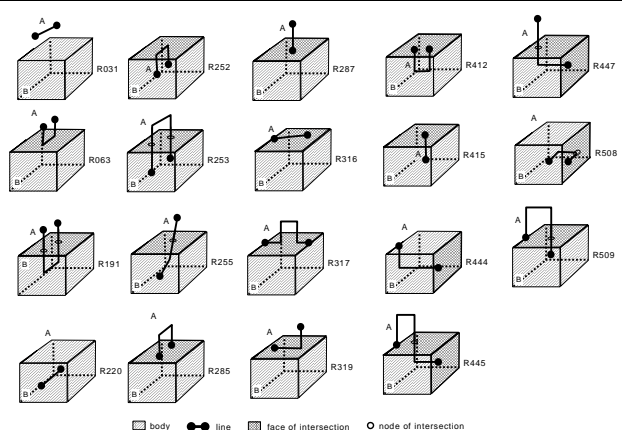


Figure 8: Body and linestring in IR 3 : 19 relationships

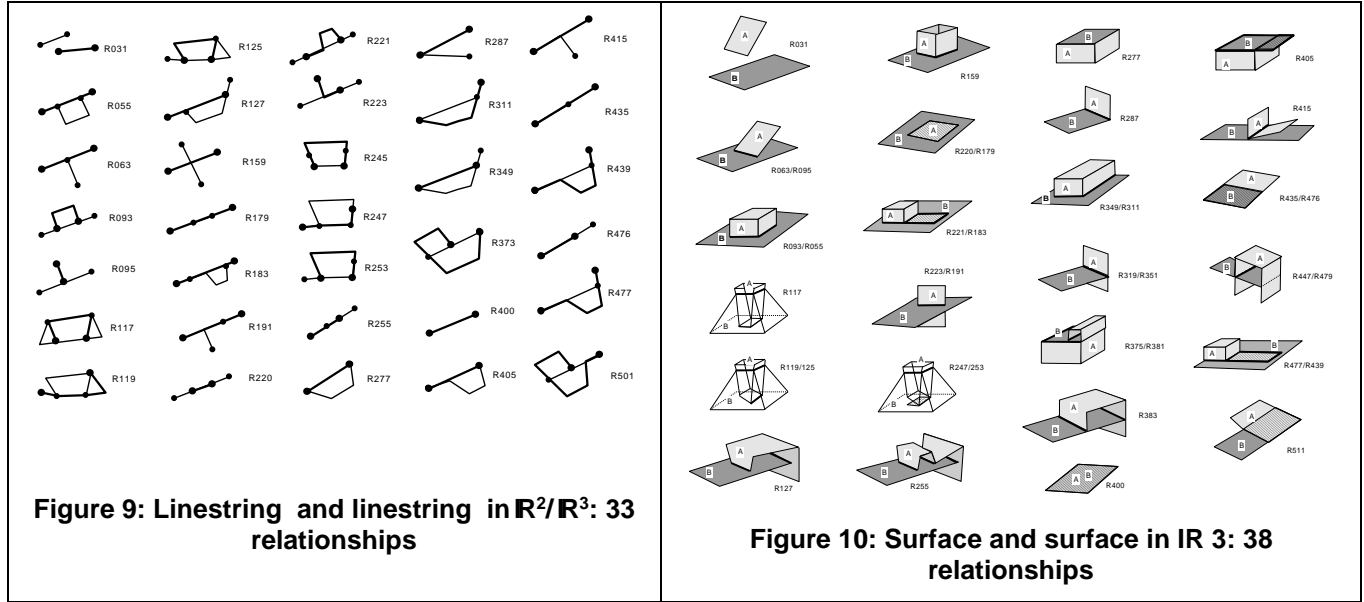


Figure 3 and Figure 4 portray the relationships between objects with the same dimension and zero co-dimension. The number of identified possible relationships are given the names: *disjoint*, *contains*, *inside*, *equal*, *touch*, *covers*, *coveredBy*, *overlap*. These relationships are also the once detected by 4IM. The relationships between other objects are given in Figure 5-9.

Although the number of relationships is significantly reduced, the question if the all have to be implemented for real-world cases remains. As it can be seen from the drawings, some of the configurations are quite complex and seem impractical. Some observations on the possible relationships are summarized below:

- Clearly the 9IM is superior to 4IM. However the increased relationships are only between object of mixed dimensions and objects of the same dimension, which are embedded in higher dimension space such as  $R(L,L)$  and  $R(S,S)$  in  $\mathbb{R}^3$ . The relationships that appear between all objects are only seven (R031, R179, R220, R400, R287, R435 and R476) and they can be detected with 4IM (Figure 3, Figure 4, Figure 5). Note that ‘overlap’ relationship R511 is not possible for  $R(L,L)$  in  $\mathbb{R}$ . The relationships that visually represents ‘overlap’ is R255. Because the boundary of linestring is disconnected, the intersection of boundaries is non-empty, i.e.  $\partial A \cap \partial B = \emptyset$  (Figure 3).
- The relationships are related to the dimension of objects, i.e. some of the relationships never occur between particular objects. For example, R509 (the interiors can intersect without touching the boundary) appears only in  $R(L,B)/R(B,L)$  and  $R(S,B)/R(B,S)$  or R277 (boundaries completely overlap but the interiors not) is possible only for  $R(L,L)$  and  $R(S,S)$  in higher dimension space. This implies that certain relationships (respectively topological operators) will be quite specific and will depend on the dimension of the objects and the space.
- The relationships are related to the geometric representation of the objects. This is to say that some relationships may not be needed because some configurations of objects are too complex and normally they are simplified. For example R455 performed for body and surface (Figure 7) may never be needed for urban applications. Most probably the surface A will be represented as two surfaces: one on the top and one inside the body B.
- As it can be realized, the names (given in Table 2) established for relationships according 4IM are not sufficient for 9IM. The relationships between surfaces in 3D space are one typical example. The human perception of ‘intersection’, ‘cross’ might be completely different: e.g. overlap stands for R511 (e.g. surface and surface in 2D) and R255 (e.g. surface and linestring in 3D). Most of the relationships are not associated with appropriate names and even it is difficult to specify the type of interaction. Further implementations topological operators would require establishment of new names.

As mentioned previously, the calculations of intersections will give equivalent results if using classical geometric representations (Figure 1). In 2011, the OGC Implementation specifications for Geographical features have been adapted



to incorporate the Dimensionality Extended Nine-Intersection model (DE-9IM) as presented by [Strobl, 2008]. The matrix for representing the 9IM is augmented to include the dimension of the intersection between the two objects as follows [Herring, 2011]:

$$R(A, B) = \begin{pmatrix} \dim(A^\circ \cap B^\circ) & \dim(A^\circ \cap \partial B) & \dim(A^\circ \cap B^-) \\ \dim(\partial A \cap B^\circ) & \dim(\partial A \cap \partial B) & \dim(\partial A \cap B^-) \\ \dim(A^- \cap B^\circ) & \dim(A^- \cap \partial B) & \dim(A^- \cap B^-) \end{pmatrix}$$

The  $\dim(x)$  gives the maximum dimension (-1, 0, 1, or 2) of the objects in the intersection  $x$ , with a numeric value of -1 corresponding to  $\dim(\emptyset)$ . The values 0, 1 and 2 correspond to point, linestring and surface intersection. For implementation, it is recommended considering a set of nine pattern-values ( $p$ ). The possible pattern values of  $p$  are {T, F, \*, 0, 1, 2}. The meaning of the values have to be as follows [Herring, 2011]:

$p = T \Rightarrow \dim(x) \in \{0, 1, 2\}$ , i.e.  $x \neq \emptyset$   
 $p = F \Rightarrow \dim(x) = -1$ , i.e.  $x = \emptyset$   
 $p = * \Rightarrow \dim(x) \in \{-1, 0, 1, 2\}$ , i.e. Don't Care  
 $p = 0 \Rightarrow \dim(x) = 0$  (point)  
 $p = 1 \Rightarrow \dim(x) = 1$  (line sting)  
 $p = 2 \Rightarrow \dim(x) = 2$  (polygon)

The OGC specifications provide further instructions how the functions have to be implemented for point, linestring, polygon and area in 2D space.

## Key applications

The topological operators are commonly implemented for vector models (boundary representation) and more specifically on geometrical models. Following the OGC Abstract specification, Topic 1 Feature Geometry, the geometry is represented as an ordered sequence of vertices that are connected by straight line segments or circular arcs, which are used to describe 0,1,2 and 3D objects. The vertices can be given with different dimensions (1D, 2D and 3D). For example, three-dimensional points are elements composed of three ordinates, X, Y and Z. Line strings are composed of one or more pairs of points that define line segments. Polygons are composed of connected line strings that form a closed ring and the area of the polygon is implied. Self-crossing polygons are not supported, although self-crossing line strings might be supported. If a line string crosses itself, it does not become a polygon and does not have any implied area.

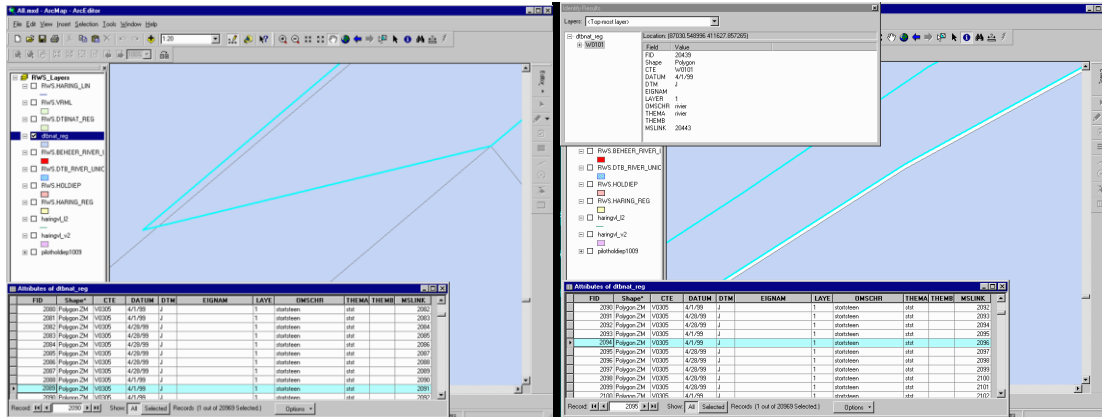
The usage of the topological relations is well-represented in query languages. All database management system with a spatial support, have provided topological operations based on the 9IM. The operations are developed by referencing the name of the 8 relationships (covers, coveredBy, contains, inside, overlaps, touch, equal, anyinteract). The operator anyinteract detects if two objects are disjoint. If the operator returns non-empty value, other operators can clarify what the relationships is. Oracle Spatial 11g, provides operators to check the topological operators in two versions as one operator SDO\_RELATE (with mask for each relationship) and as convenience operators named after the relationships. PostGIS provides operators per relationship.

## Consistency and validity

The topological operations are utilised for different purposes. The most common use is investigation if a geographical map is a planar partition, i.e. to detect gaps or overlaps between geographical objects. These operations are of major importance for organisations, which maintain topographic, land use, cadastral, hydrological, etc. maps, such as National mapping organisations, Cadastre, Topographic offices, Municipalities, etc.

An example of the use of Oracle Spatial operation and function for the work of Rijkswaterstaat, the Dutch organisation responsible for design, construction, management and maintenance of infrastructure facilities is presented in [Zlatanova, Tijssen, van Oosterom, et al, 2003]. The main goal of the experiments was the possibility to perform validity check and consistent maintenance at database level utilising the existing operators. An overlapping test was performed to investigate the relationships between the polygons in one map layer (assuming polygons should not overlap). All the datasets were checked (with the operator SDO\_RELATE) for the relationship ANYINTERACT. If the operator detects interaction, the

geometry of the overlapping polygon is composed using the Oracle Spatial and Graph function SDO\_GEOM.SDO\_INTERSECTION. The function returns an object that is the topological intersection (AND operation) of two geographical objects. Finally, the area of the intersecting polygon is computed using again the Oracle Spatial and Graph function, SDO\_GEOM.SDO\_AREA. The computed area gives an indication over the type of overlap. The area of all computed polygons should be equal to the area of the outer polygon of the given area. Following this procedure ‘overlaps’ can be easily detected.



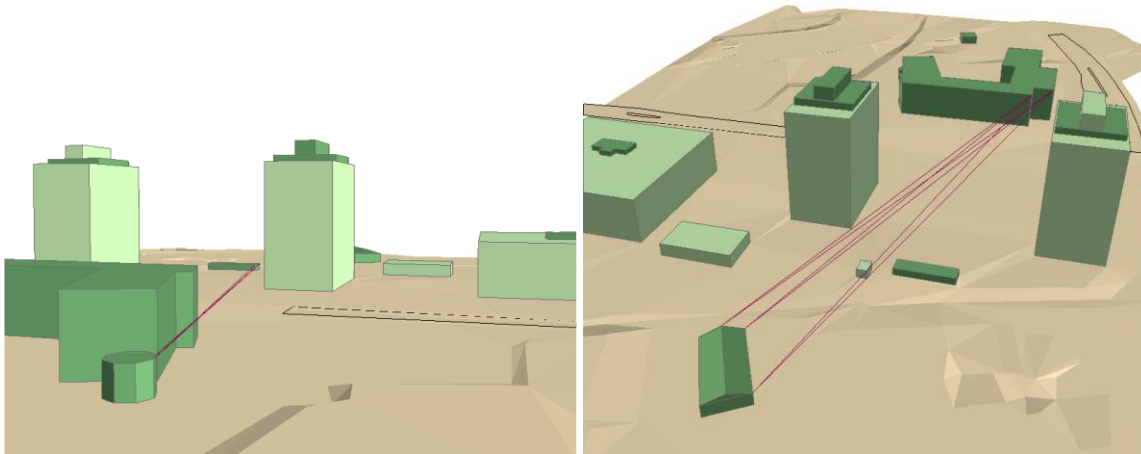
**Figure 11: Detecting overlaps and gaps between two polygons with SDO\_RELATE (mask ‘ANYINTERACT’)**

It should be noticed that most of the topological operators are currently implemented for a 2D space. If an object is given with its 3D coordinates, it is first projected on the xy-plane and then the operation is performed. The topological operators might even trigger an error if vertical objects are given as predicates. Oracle Spatial and Graph 11g provides few operators such as SDO\_ANYINTERACT and SDO\_INSIDE, which make use of the 3D coordinates.

The topological operations can be used to define constraints either for update of existing data sets, i.e. to avoid wrong overlaps and intersections between geographical object, or to restrict the use from performing certain actions, which will result in violation of semantic or topological rules. [Lowsma, Zlatanova, van Lammeren et al 2006] present such a set of constraints, which are implemented in a landscape simulator SALIX-2. Examples of a constraint on the topological relationships between two objects are ‘Yucca tree must never stand in water’ and ‘the tree must never be planted on a paving’. These constraints are then checked each time when students plant trees in the landscape simulator. SALIX-2 maintains trees, bushes and ground surfaces (water, paving, soft paving, grass, and bridge). The constraints are implemented in Oracle Spatial 9i using triggers (before and after) and the operators SDO\_ANYINTERACT and SDO\_RELATE (mask = ‘inside’).

## Visibility analysis

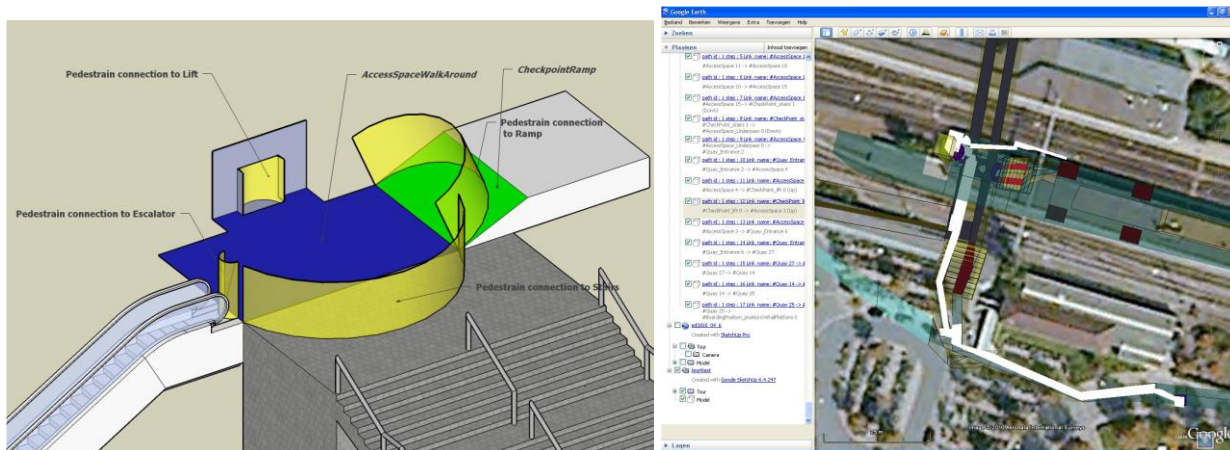
Although most of the topological operators are 2D, they can be readily applied to 2.5D objects, i.e. object that have only one z-coordinate for a given x,y. The relationships between a large number of 3D geographic objects can also be derived on the basis of 2D investigations. For example most of the topological relationships between city models in LOD1 or LOD2 are easy to check using the 2D operators. Only relationship ‘inside’ would require clarification of the intersection in Z-direction. Such analysis can be performed by developing database scripts. Scripting, which use 2D topological operations is a power approach to complex spatial analysis, e.g. visibility line. Figure 12 illustrates visibility analysis between buildings, which is computed entirely with PostGIS operations and user defined functions, which make use of the operator ST\_intersect.



**Figure 12 Visibility analysis, utilizing ST\_Intersects in PostGIS (courtesy Martin Warmer).**

### Navigation

Topological relationships can be used to define surfaces, appropriate for walking and support path finding on the basis of adjacent patches [Schaap, Zlatanova & van Oosterom, 2012]. The patches are defined in such a way to slightly overlap. These overlapping parts are lately used to find possible directions for movement. A 3D model is proposed, which maintains information of a network and set of surface patches with corresponding attributes and behaviour. The model is implemented in Oracle 11g and results are in Google Earth. Oracle Spatial and Graph 11g network shortest route capabilities for logical network are used for finding the optimal route for individual travellers with specific speed and accessibility properties. The application makes use of the Oracle Spatial and Graph operations SDO\_ANYINTERACT and SDO\_RELATE to define the walking surfaces (Figure 13)

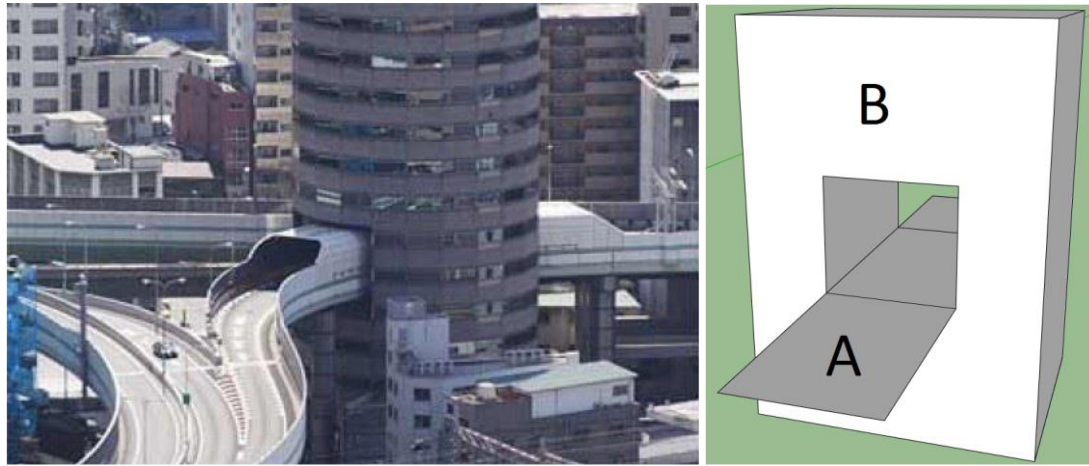


**Figure 13: Overlapping patches for navigation and the resulting walking route (SDO\_INTERACT and SDO\_RELATE).**

### 3D topological relationships

There are still many cases that true 3D operator are needed. 3D world contains more information than 2D and the validity and integrity of 3D models are getting of increasing importance (Figure 14) [Ellul, 2013]. [Borrman & Rank, 2009] present algorithms define to *inside*, *contain*, *touch*, *overlap*, *disjoint* and *equal* in 3D space for the purpose of analysing Building information models. The operators are implemented in 3D raster domain (octree-based), i.e. prior performing the operations the boundary objects is voxelised.

[Xu & Zlatanova, 2013] argues that topological operations have to be advanced to 3D by investigating which real-world object can result in 3D conflicting situations. They concentrate on the objects as defined within CityGML and propose three steps: 1) investigating the semantics of the objects and expressing the impossible relationships in natural language ('houses can never intersect at any LOD', 'trees can intersect at their branches'), 2) translating to geometric representations ('bodies cannot intersect if houses', 'surfaces can intersect if trees' ) and 3) defining the needed 3D relationship ( $R(B,B) = \{ R031, R278 \}$  and  $R(S,S) = \{ R287, R511 \}$ ). Most of the 3D operations are needed for man-made objects (such as buildings, tunnels, bridges) and trees. Relationships between buildings, tunnels and bridges would require considering geometries with holes. Several relationships are then implemented, re-using 2D topological operators.



**Figure 14: Examples of relationships between objects, which cannot be detected by 2D topological operators [Xu & Zlatanova 2013]**

## Future directions

Advances in data collection, processing, management and visualisation have led to and increased need for well-defined objects, consist data sets and elaborated analysis. The size of current data sets and complexity of 3D models have increased the interest in database storage with centralized management of information with extended analytical capabilities. The topology and topological relationships are key component in this process. Future investigations and developments are envisaged under the following broad groups: algorithms, data structures and applications.

Topological relationships are most well-developed for simple objects and implemented for 2D objects. Next step is the implementations of the DE-9IM for 2D objects. Algorithms for 1D, 2D and 3D objects embedded in 3D space are still investigated only in research environment. A first option of extending the operations in 3D is by re-using and combining existing operators. The complexity of real world man-made objects clearly shows that simple objects with holes and tunnels have to be urgently included in research agendas. A small set of 3D operators on solids with tunnels could be the first easy step. Investigations in higher order topology, which is beyond the immediate neighbours or considering semantic expressions is very much of interest.

The implementation of topological relationships is highly depended on the underlying spatial model. Current implementations make use of geometrical model as defined in the OGC abstract specification. Research and prototype implementations have convincingly shown that topological models have advantages to the geometrical model in keeping consistency and especially facilitating adjacency relationships. The disadvantage of topological data structures is the lack of unified data structure in 3D. [van Oosterom, Stoter, Quak et al 2002] propose maintenance of the existing topological data structures in a metadata table similar to the reference coordinate systems. A complete and detailed description of all parameters defining topological model such as dimension, primitives used, existence of explicit relationships, number of tables, rules, etc., will make possible to transform data between topological models.

The topological operations are presently discussed only for vector models. However they can be implemented for 2D or 3D raster models as well. The topological primitives interior, boundary and exterior remain the same but their importance

changes. In vector models, the boundary is the most critical primitive, which is realised through the shape of the object. In raster/voxel domain, the interior will be the leading primitive. Considering the complexity of 3D objects, 3D raster domain with its unified primitives (voxels) and well-defined structures (e.g. octree) can be an attractive option for implementing 3D topological operations.

Topological operations have been traditionally developed to represent relationships between geographical objects. The interest in topology in more domains is increasing. One straight-forward implementation is detecting connectivity between objects and deriving dual graph. This approach has been widely accepted for shortest path computations, spatial syntax analysis and walk through. Recently a new standard IndoorGML has been adopted by OGC, which utilises the prima-dual approach for indoor navigation [Lee, Li, Zlatanova et al 2014]. Topological relationships attract the attention of CAD and BIM domain as well. Indoor environments are truly 3D, contain much more objects with multifaceted shapes and complex relationships. Examples are pipes and cables in buildings, furniture in rooms, elevators and escalators between floors. Such relationships will definitely need extended 3D topological relationships. The richer set of geometric primitives in CAD and BIM domains, the density of objects and complexity of relationships could be another reason for shift to the 3D raster domain as unified environment for 3D topological relationships.

## **Cross references**

### **Modelling of Spatial relations**

#### **Data Structure**

#### **Graph Theory, Koningsberg problem**

#### **Mathematical foundations of GIS**

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**Table 1: Negative conditions for eliminating impossible relationships in  $\mathbb{R}$ ,  $\mathbb{R}^2$  and  $\mathbb{R}^3$**

Group	$R(A, B)$	$\partial A \cap \partial B$	$A^\circ \cap B^\circ$	$\partial A \cap B^\circ$	$A^\circ \cap \partial B$	$A^- \cap B^-$	$A^- \cap \partial B$	$A^\circ \cap B^\circ$	$\partial A \cap B^-$	$A^\circ \cap B^-$
1	C1	-	-	-	-	0	-	-	-	-
	C2 <sub>a</sub>	-	-	-	-	-	-	-	1	0
	C2 <sub>b</sub>	-	-	-	-	-	1	0	-	-
	C3 <sub>a</sub>	0	-	0	-	-	-	-	0	-
	C3 <sub>b</sub>	0	-	-	0	-	0	-	-	-
2	C4 <sub>a</sub>	-	0	-	-	-	-	-	-	0
	C4 <sub>b</sub>	-	0	-	-	-	-	0	-	-
	C5 <sub>a</sub>	-	-	-	1	-	-	-	-	0
	C5 <sub>b</sub>	-	-	1	-	-	-	0	-	-
3	C6 <sub>a</sub>	-	-	-	-	-	-	-	-	0
	C6 <sub>b</sub>	-	-	-	-	-	-	-	0	-
	C6 <sub>c</sub>	-	-	-	-	-	0	-	-	-
	C6 <sub>d</sub>	-	-	-	-	-	-	0	-	-
4	C7 <sub>a</sub>	-	0	-	0	-	-	-	-	0
	C7 <sub>b</sub>	-	0	0	-	-	-	0	-	-
5	C8 <sub>a</sub>	-	0	1	-	-	-	-	-	-
	C8 <sub>b</sub>	-	0	-	1	-	-	-	-	-
	C9 <sub>a</sub>	-	1	-	0	-	-	-	-	1
	C9 <sub>b</sub>	-	1	0	-	-	-	1	-	-
6	C10 <sub>a</sub>	1	-	-	1	-	1	-	-	-
	C10 <sub>b</sub>	1	-	1	-	-	-	-	1	-
7	C11 <sub>a</sub>	0	-	1	-	-	-	-	1	-
	C11 <sub>b</sub>	0	-	-	1	-	1	-	-	-
8	C12	0	-	-	-	-	0	-	0	-
	C13	-	0	-	-	-	0	-	0	-
	C14 <sub>a</sub>	-	-	-	-	-	-	-	0	1
	C14 <sub>b</sub>	-	-	-	-	-	0	1	-	-
9	C15 <sub>a</sub>	-	-	-	-	-	0	1	-	0
	C15 <sub>b</sub>	-	-	-	-	-	-	0	0	1
	C16 <sub>a</sub>	-	-	0	1	-	-	-	0	-
	C16 <sub>b</sub>	-	-	1	0	-	0	-	-	-
10	C17 <sub>a</sub>	-	-	0	0	-	1	-	0	-
	C17 <sub>b</sub>	-	-	0	0	-	0	-	1	-
	C18 <sub>a</sub>	-	1	1	1	-	0	-	0	-
	C18 <sub>b</sub>	1	-	1	1	-	0	-	0	-
	C19 <sub>a</sub>	1	1	1	1	-	-	-	0	-
	C19 <sub>b</sub>	1	1	1	1	-	0	-	-	-
11	C20 <sub>a</sub>	-	0	-	1	-	-	0	-	-
	C20 <sub>b</sub>	-	0	1	-	-	-	-	-	0
	C21 <sub>a</sub>	1	0	0	-	-	-	0	-	-
	C21 <sub>b</sub>	1	0	-	0	-	-	-	-	0
12	C22 <sub>a</sub>	1	-	0	-	-	1	-	0	-
	C22 <sub>b</sub>	1	-	-	0	-	0	-	1	-
13	C23 <sub>a</sub>	1	-	-	-	-	-	-	-	-
	C23 <sub>b</sub>	-	-	1	-	-	-	-	-	-
	C23 <sub>c</sub>	-	-	-	-	-	-	-	1	-
	C23 <sub>d</sub>	-	-	-	1	-	-	-	-	-
	C23 <sub>e</sub>	-	-	-	-	-	1	-	-	-
	C24 <sub>a</sub>	-	1	-	1	-	-	-	-	-
	C24 <sub>b</sub>	-	1	-	-	-	-	-	-	1
	C24 <sub>c</sub>	-	-	-	1	-	-	-	-	1
	C24 <sub>d</sub>	-	1	1	-	-	-	-	-	-
	C24 <sub>e</sub>	-	1	-	-	-	-	1	-	-
	C24 <sub>f</sub>	-	-	1	-	-	-	1	-	-
	C25 <sub>a</sub>	-	0	-	-	-	-	0	-	-
	C25 <sub>b</sub>	-	0	-	-	-	-	-	-	0



**Table 2: Possible relationships between spatial objects in  $\mathbb{R}$ ,  $\mathbb{R}^2$  and  $\mathbb{R}^3$**

		$R(A,B)$	$\partial A \cap \partial B$	$A^\circ \cap B^\circ$	$\partial A \cap B^\circ$	$A^\circ \cap \partial B$	$A^- \cap B^-$	$A^- \cap \partial B$	$A^\circ \cap B^\circ$	$\partial A \cap B^-$	$A^\circ \cap B^-$	
1	Disjoint	R026	0	0	0	0	1	1	0	1	0	R026
		R027	0	0	0	0	1	1	0	1	1	R027
		R030	0	0	0	0	1	1	1	1	0	R030
		R031	0	0	0	0	1	1	1	1	1	R031
2		R051	0	0	0	1	1	0	0	1	1	R051
		R055	0	0	0	1	1	0	1	1	1	R055
		R063	0	0	0	1	1	1	1	1	1	R063
3		R092	0	0	1	0	1	1	1	0	0	R092
		R093	0	0	1	0	1	1	1	0	1	R093
		R095	0	0	1	0	1	1	1	1	1	R095
4		R117	0	0	1	1	1	0	1	0	1	R117
		R119	0	0	1	1	1	0	1	1	1	R119
		R125	0	0	1	1	1	1	1	0	1	R125
		R127	0	0	1	1	1	1	1	1	1	R127
5	Crosses	R159	0	1	0	0	1	1	1	1	1	R159
6	Contains	R179	0	1	0	1	1	0	0	1	1	R179
		R183	0	1	0	1	1	0	1	1	1	R183
		R191	0	1	0	1	1	1	1	1	1	R191
7	Inside	R220	0	1	1	0	1	1	1	0	0	R220
		R221	0	1	1	0	1	1	1	0	1	R221
		R223	0	1	1	0	1	1	1	1	1	R223
8	Crosses?	R243	0	1	1	1	1	0	0	1	1	R243
		R245	0	1	1	1	1	0	1	0	1	R245
		R247	0	1	1	1	1	0	1	1	1	R247
		R252	0	1	1	1	1	1	1	0	0	R252
		R253	0	1	1	1	1	1	1	0	1	R253
		R255	0	1	1	1	1	1	1	1	1	R255
9	Touch	R272	1	0	0	0	1	0	0	0	0	R272
		R275	1	0	0	0	1	0	0	1	1	R275
		R277	1	0	0	0	1	0	1	0	1	R277
		R279	1	0	0	0	1	0	1	1	1	R279
		R284	1	0	0	0	1	1	1	0	0	R284
		R285	1	0	0	0	1	1	1	0	1	R285

